EEC 603
MICROWAVE ENGINEERING

UNIT-1

TEM WAVES

$$\nabla_t \bullet \bar{h}_t = 0 \quad , \quad \nabla_t \bullet \bar{e}_t = 0$$

$$\nabla_t \times \bar{e}_t = 0 \quad , \quad \beta \hat{a}_z \times \bar{e}_t = \omega \mu_0 \bar{h}_t$$

$$\nabla_t \times \bar{h}_t = 0 \quad , \quad \beta \hat{a}_z \times \bar{h}_t = -\omega \varepsilon_0 \bar{e}_t$$

$$\bar{e}(x, y) = -\nabla_t \Phi(x, y) = 0 \qquad \Phi \equiv \text{Scalar Potential}$$

$$\therefore \nabla_t^{\,2} \Phi(x, y) = 0$$

$$\bar{E}_t = \bar{e}_t e^{\mp j\beta z} = -\nabla_t \Phi(x, y) e^{\mp j\beta z}$$

$$\bar{H}_t = \pm \bar{h}_t e^{\mp j\beta z} = \pm Y_0 \hat{a}_z \times \bar{e}\, e^{\mp j\beta z}$$

$$Y_0 = \sqrt{\frac{\varepsilon}{\mu}} = \frac{1}{Z_0} \quad , \qquad \eta_0 = \text{Wave Impedance}$$

$$\frac{\mathrm{E_x}}{\mathrm{H_y}} = -\frac{E_y}{H_x} = \pm\eta_0$$

$\pm$ for wave propagation in the + or - z direction

The field must satisfy Helmholtz equation :

$$\nabla^2 E_t + k_0^2 E_t = 0 \quad , \quad \text{but } \nabla = \nabla_{\mathrm{t}} - j\bar{\beta} a_z \qquad , \nabla^2 = \nabla_t^2 - \beta^2$$

$$\nabla_t^2 \overline{E}_t + (k_0^2 - \beta)\overline{E}_t = 0 \qquad , \qquad \nabla_{\mathrm{t}} [\nabla_t^2 \phi + (k_0^2 - \beta)\phi] = 0$$

$\beta = \pm k_0$ for TEM waves

TE WAVES

$$\nabla^2 \overline{H} + k^2 \overline{H} = 0$$

$$(\nabla_t{}^2 - \beta^2) h_z(x, y) + k^2 h_z = 0$$

$$\nabla_t{}^2 + (k^2 - \beta^2) h_z = 0, \qquad \text{let } k_c^2 = k^2 - \beta^2$$

$$\nabla_t{}^2 h_z + k_c^2 h_z = 0$$

$$\nabla_t \times \overline{e}_t = -j\omega\overline{\mu} h_z \qquad , \qquad \beta\overline{a}_z \times \overline{e}_t = \omega\mu\overline{h}_t$$

$$\nabla_t \times \overline{h}_t = 0 \qquad , \qquad \overline{a}_z \times \nabla_t h_z + j\beta a_z \times \overline{h}_t = -j\omega\varepsilon\overline{e}$$

$$\nabla_t \bullet \overline{h}_t = j\beta h_z \qquad , \qquad \nabla_t \bullet \overline{e}_t = 0$$

$$\therefore \overline{h}_t = -\frac{j\beta}{k_c^2} \nabla_t h_z$$

$$\overline{e}_t = -\frac{\omega\mu_0}{\beta} \hat{a}_z \times \overline{h}_t = -\frac{k}{\beta} Z_0 \hat{a}_z \times \overline{h}_t \quad ; \quad \eta_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

$$Z_h = \frac{k}{\beta} \eta_0 \equiv \text{Wave Impedance}$$

$$\frac{\mathrm{e}_{\mathrm{x}}}{\mathrm{h}_{\mathrm{y}}} = -\frac{e_y}{h_x} = Z_h$$

TM WAVES

$$\nabla^2 \overline{E} + k^2 \overline{E} = 0$$

$$(\nabla_t^{\,2} - \beta^2) e_z(x, y) + k^2 e_z = 0$$

$$\nabla_t^{\,2} e_z + (k^2 - \beta^2) e_z = 0, \qquad \text{let } k_c^2 = k^2 - \beta^2$$

$$\nabla_t^{\,2} e_z + k_c^2 e_z = 0$$

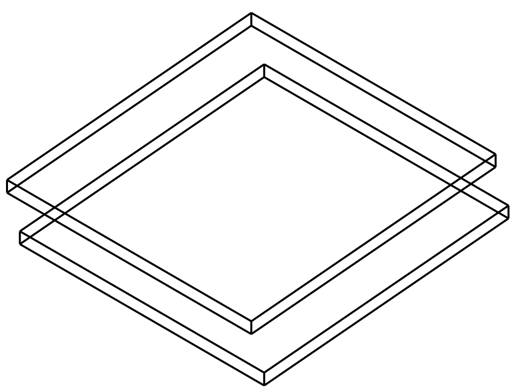
$$\therefore \overline{e}_t = -\frac{j\beta}{k_c^2} \nabla_t e_z$$

$$\overline{h}_t = \pm Y_e \hat{a}_z \times \overline{e}$$

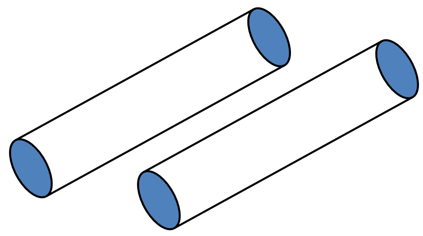
$$Y_e = \frac{k}{\beta} Y_0 \equiv \text{Wave Admittance}$$

TEM TRANSMISSION LINES

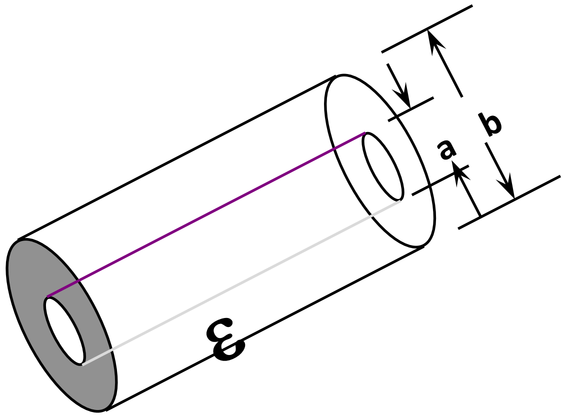
Parallel -plate



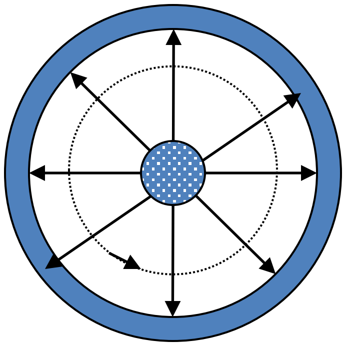
Two-wire

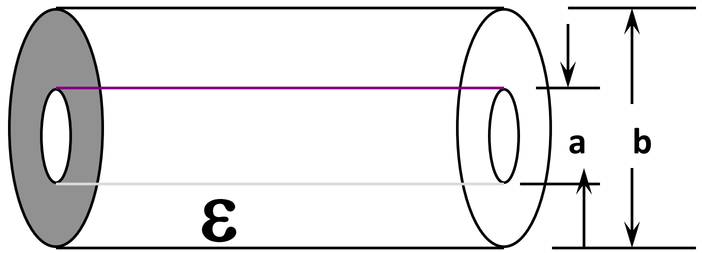


Coaxial



COAXIAL LINES





$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \Phi^2}=0 \qquad \text{for } \frac{\partial}{\partial \Phi}=0$$

$$\Phi = C_1 \ln r + C_2 \qquad \Phi = V_0 \text{ at } r = a, \Phi = 0 \text{ at } r = 0$$

$$\Phi = V_0 \frac{\ln(r/b)}{\ln(a/b)}$$

$$\overline{E} = \frac{V_0}{r\ln(b/a)}\,\overline{\mathrm{a}}_{\mathrm{r}}\;\mathrm{e}^{-\mathrm{jkz}} \qquad \text{and} \qquad H_{\varphi} = Y_0\frac{V_0}{r\ln(b/a)}\,\overline{\mathrm{a}}_{\phi}\;\mathrm{e}^{-\mathrm{jkz}}$$

$$Y_0 = \sqrt{\frac{\varepsilon}{\mu}}$$

$$\bar{J}_s = \hat{n} \times \bar{H} = \hat{a}_r \times \bar{H} = Y_0 \frac{V_0}{a \ln(b/a)} \hat{a}_z \mathrm{e}^{-\mathrm{jkz}}$$

$$\mathrm{I} = Y_0 \frac{V_0}{a \ln(b/a)} \int_0^{2\pi} a d\phi \mathrm{e}^{-\mathrm{jkz}} = Y_0 \frac{2\pi V_0}{\ln(b/a)} \mathrm{e}^{-\mathrm{jkz}}$$

$$P = \frac{1}{2} \mathrm{Re} \int_a^b \int_0^{2\pi} \bar{E} \times \bar{H}^* \bullet \hat{a}_z r dr d\phi = \frac{\pi Y_0 V_0^2}{\ln(b/a)}$$

- **THE CHARACTERISTIC IMPEDANCE OF A COAXIAL IS Z_0**

$$Z_c = \frac{V_0}{I_0} = \frac{1}{2\pi Y_0} \ln\left(\frac{b}{a}\right) \quad \text{Ohms}$$

TM modes

$$\left(\frac{\partial^2}{\partial y^2} + k_c^2\right) e_z(x,y) = 0$$

$$e_z(x,y) = A\sin k_c y + B\cos k_c y$$

$$e_z(x,y) = 0 \quad \text{at y} = 0, \text{d}$$

$$\text{B} = 0 \qquad , \; k_c d = n\pi \;\; , n = 0,1,2,3,....$$

$$\beta = \sqrt{K^2 - (\frac{n\pi}{d})^2}$$

$$e_z(x,y) = A_n \sin\frac{n\pi}{d} y$$

$$E_z(x,y,z) = A_n \sin\frac{n\pi}{d} y \;\; \mathrm{e}^{-\mathrm{j}\beta \mathrm{z}}$$

$$H_x(x,y,z) = \frac{j\omega\varepsilon}{k_c} A_n \cos\frac{n\pi}{d} y \;\; \mathrm{e}^{-\mathrm{j}\beta \mathrm{z}}$$

$$E_y(x,y,z) = \frac{-j\beta}{k_c} A_n \cos\frac{n\pi}{d} y \; e^{-j\beta z}, \quad E_x = H_y = 0$$

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{\eta}{2d\sqrt{\mu\varepsilon}}$$

The wave impedance of the TM modes is :

$$Z_{TM} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\varepsilon} = \frac{\beta\eta}{k}$$

$$v_p = \frac{\omega}{\beta} \qquad , \qquad \lambda_g = \frac{2\pi}{\beta}$$

$$P_0 = \frac{1}{2}\int_{x=0}^{w}\int_{y=0}^{d} \overline{E} \times \overline{H}^* \bullet \hat{z}\,dydx = -\frac{1}{2}\int_{x=0}^{w}\int_{y=0}^{d} E_y H_x^* dydx$$

$$= \begin{cases} \dfrac{\omega \operatorname{Re}(\beta)\omega\varepsilon d}{4k_c^2}\left|A_n\right|^2 & \text{for n} > 0 \\ \dfrac{\omega \operatorname{Re}(\beta)\omega\varepsilon d}{2k_c^2}\left|A_n\right|^2 & \text{for n} > 0 \end{cases}$$

Attenuation due to conductor loss

$$\alpha_{\mathrm{c}} = \frac{P_\ell}{2P_0}$$

$$P_\ell = 2\left(\frac{R_s}{2}\right)\int_{x=0}^{w}\left|\bar{J}_s\right|^2 dx = \frac{\omega^2\varepsilon^2 R_s w}{k_c^2}\left|A_n\right|^2$$

$$\alpha_c = \frac{2\omega R_s}{\beta d} = \frac{2kR_s}{\beta\eta d}\ \ \mathrm{Np/m}\quad \text{for n} > 0$$

TE Modes

$$\left(\frac{\partial^2}{\partial y^2} + k_c^2\right) h_z(x, y) = 0$$

$$h_z(x, y) = A \sin k_c y + B \cos k_c y$$

$$e_x(x, y) = 0 \quad \text{at y} = 0, \text{d}$$

$$\text{A} = 0 \qquad , \; k_c d = n\pi \; , n = 1,2,3,....$$

$$\beta = \sqrt{k^2 - (\frac{n\pi}{d})^2}$$

$$h_z(x, y) = B_n \cos \frac{n\pi}{d} y$$

$$H_z(x, y, z) = B_n \cos \frac{n\pi}{d} y \;\; \text{e}^{-\text{j}\beta\text{z}}$$

$$E_x(x, y, z) = \frac{j\omega\mu}{k_c} B_n \sin \frac{n\pi}{d} y \;\; \text{e}^{-\text{j}\beta\text{z}}$$

$$H_y(x,y,z) = \frac{j\beta}{k_c} B_n \sin\frac{n\pi}{d} y \ \mathrm{e}^{-j\beta z}, \quad \mathrm{E_y} = H_x = 0$$

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{\eta}{2d\sqrt{\mu\varepsilon}}$$

The wave impedance of the TM modes is :

$$\mathrm{Z_{TE}} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$$

$$v_p = \frac{\omega}{\beta} \qquad , \qquad \lambda_{\mathrm{g}} = \frac{2\pi}{\beta}$$

$$P_0 = \frac{1}{2}\int_{x=0}^{w}\int_{y=0}^{d} \overline{E} \times \overline{H}^* \bullet \hat{z}\,dydx = \frac{1}{2}\int_{x=0}^{w}\int_{y=0}^{d} E_x H_y^* dydx$$

$$= \frac{\omega\mu dw}{4k_c^2}\left|B_n\right|^2 \mathrm{Re}(\beta) \qquad \text{For n} > 0$$

$$\alpha_{\mathrm{c}} = \frac{2k_c^2 R_s}{k\beta\eta d} \quad \text{Np/m}$$